International Journal For Recent Developments in Science & Technology
2581-4575
Crossref
A Peer Reviewed Research Journal

Adomian Decomposition Method for Korteweg- deVries Fifth Order Equation

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Abstract

In this paper, behavior of solution of Korteweg-de Vries (KdV) nonlinear partial differential equation(NPD) [2]of fifth order is discussed .The Adomian decomposition method is implemented for finding solution of fifth order (fKdV).Solutions obtain from this method is compare with exact solution of fKdV equation. Soluion obtained by ADM has better agreement with its exact solution. Also this solution is represented graphically.

Keywords: Adomian Decomposition Method, Adomian polynomials, fifth order Korteweg-deVries equation, nonlinear Partial differential equation,exact solution.

1. Introduction:

In recent years, The Adomian Decomposition Method (ADM) has more attention of researchers in applied science. The ADM was first introduced by George Adomian in 1980.This method is very powerful and easily applies to linear, nonlinear ,ordinary and partial differential equations.There are many nonlinear partial differential equation occurs in Engineering and Science field. The fKdV nonlinear partial differential equation has been studied in the theory of shallow water waves with surface tension and in theory of magneto acoustic waves in plasmas.

In this paper, ADM is implemented for solving fKdV nonlinear partial differential equation.The Paper has been organized as follows: In section 2, ADM for NPD equations is introduced. Adomian Polynomials is also discussed in this section. In section 3, Solution of Nonlinear Partial Differential fKdV by using ADM is analyzed. The last section deals with conclusion.

2.ADM for NPD equations:

In this section nonlinear terms occurs in NPD equation can be express in infinite series.

Consider NPD equation,

$$Dv + Rv + Nv = g \qquad (1)$$

Here Nv is nonlinear term like $v^2, v^3, v^4, vv_x, e^v, sinv, v_x^2$, etc. D is derivative operator which is invertible,$Rv(x,t)$is a linear function and g is source term.

By applying D^{-1} to both sides of equation (1), we get

$$v = h - D^{-1}(Rv + Nv \tag{2}$$

where the function h is obtained by integrating the term g and using given constrains. Also as similar to previous by decomposing v into sum ofinfinite number of components as,

$$v = \sum_{i=0}^{\infty} v_i \tag{3}$$

here the components $v_i, i \geq 0$ can be calculated through recursive relation. The nonlinear $Nv(x,t))$ is obtained by expressing it into infinite series as,

$$Nv = \sum_{i=0}^{\infty} Ai \tag{4}$$

here Ai is Adomian polynomials which we will discuss in details in next section.

By substituting values of equation (3) and (4) in equation (2) we get,

$$\sum_{i=0}^{\infty} v_i = h - D^{-1}\left(R\sum_{i=0}^{\infty} v_i + \sum_{i=0}^{\infty} Ai\right)$$

After deriving Adomian Polynomial Ai,we can calculate $v_i(x,t)$ by using recursive relation,

$$v_0(x,t) = h,\ v_{i+1} = -D^{-1}\left(R\sum_{i=0}^{\infty} v_i + \sum_{i=0}^{\infty} Ai\right),\quad i \geq 0$$

Substituting the values of $v_i, i \geq 0$ in equation (2) we get solution of NPD equation (1) by using ADM.

Adomian Polynomials:

As seen in previous section, a nonlinear term occurs in NPD equation is represented by infinite series given by equation (4).hence polynomial $Ai,\ i \geq 0$ in equation (4) is known as Adomian polynomials. In this section we will discuss method of calculating Adomian Polynomials present in infinite series.

Determination of Adomian Polynomials:

In this method for calculating Adomian Polynomials, Adomian introduced a formula which is given as,

$$A_i = \frac{1}{i!}\frac{d^i}{d\beta^i}\left[N\left(\sum_{j=0}^{i}\beta^j v_j\right)\right]_{\beta=0}, \qquad i \geq 0 \tag{5}$$

Where $Nv(x,t)$ is a nonlinear function.by using equation (5) we can calculate Adomian Polynomials as follows,

$$A_0 = N(v_0),$$

$$A_1 = \frac{1}{1!}\frac{d}{d\beta}\left[N\left(\sum_{j=0}^{1}\beta^j v_j\right)\right]_{\beta=0}$$

$$= \frac{d}{d\beta}[N(v_0 + \beta v_1)]_{\beta=0}$$

$$= [N'(v_0 + \beta v_1)]_{\beta=0} v_1$$

$$= N'(v_0)v_1$$

$$= v_1 N'(v_0)$$

$$A_2 = \frac{1}{2!}\frac{d^2}{d\beta^2}\left[N\left(\sum_{j=0}^{2}\beta^j v_j\right)\right]_{\beta=0}$$

$$= \frac{1}{2!}\frac{d^2}{d\beta^2}[N(v_0 + \beta v_1 + \beta^2 v_2)]_{\beta=0}$$

$$= \frac{1}{2!}\frac{d}{d\beta}[N'(v_0 + \beta v_1 + \beta^2 v_2).(v_1 + 2\beta v_2)]_{\beta=0}$$

$$= \frac{1}{2!}\{[N'(v_0 + \beta v_1 + \beta^2 v_2).(2v_2)] + N''(v_0 + \beta v_1 + \beta^2 v_2).(v_1 + 2\beta v_2)^2\}_{\beta=0}$$

$$= v_2.N'(v_0) + \frac{1}{2!}{v_1}^2 N''(v_0)$$

$$A_3 = \frac{1}{3!}\frac{d^3}{d\beta^3}\left[N\left(\sum_{j=0}^{3}\beta^j v_j\right)\right]_{\beta=0}$$

$$= \frac{1}{3!}\frac{d^3}{d\beta^3}[N(v_0 + \beta v_1 + \beta^2 v_2 + \beta^3 v_3)]_{\beta=0}$$

$$= \frac{1}{3!}\frac{d^2}{d\beta^2}[N'(v_0 + \beta v_1 + \beta^2 v_2 + \beta^3 v_3)(v_1 + 2\beta v_2 + 3\beta^2 v_3)]_{\beta=0}$$

$$= \frac{1}{3!}\frac{d}{d\beta}\{[N'(v_0+\beta v_1+\beta^2 v_2+\beta^3 v_3)(2v_2+6\beta v_3)]$$
$$+[N''(v_0+\beta v_1+\beta^2 v_2+\beta^3 v_3)(v_1+2\beta v_2+3\beta^2 v_3)^2]\}_{\beta=0}$$

$$= \frac{1}{3!}\{[N'(v_0+\beta v_1+\beta^2 v_2+\beta^3 v_3).(6v_3)]$$
$$+[N''(v_0+\beta v_1+\beta^2 v_2+\beta^3 v_3)(v_1+2\beta v_2+3\beta^2 v_3).(2v_2+6\beta v_3)]$$
$$+[N''(v_0+\beta v_1+\beta^2 v_2+\beta^3 v_3).2(v_1+2\beta v_2+3\beta^2 v_3).(2v_2+6\beta v_3)]$$
$$+[N'''(v_0+\beta v_1+\beta^2 v_2+\beta^3 v_3).(v_1+2\beta v_2+3\beta^2 v_3)^3]\}_{\beta=0}$$

$$= v_3 N'(v_0) + \frac{1}{3!}\{[N''(v_0)(v_1)(2v_2)] + [N''(v_0)2(v_1)(2v_2)] + [N'''(v_0)(v_1)^3]\}$$

$$= v_3.N'(v_0) + v_1 v_2 N''(v_0) + \frac{1}{3!}{v_1}^3 N'''(v_0)$$

Similarly we can calculate other Adomian Polynomials $A_4,\ A_5,\ A_6, \ldots$

Hence we get,

$A_0 = N(v_0),$

$A_1 = v_1 N'(v_0)$

$A_2 = v_2 N'(v_0) + \frac{1}{2!}{v_1}^2 N''(v_0)$

$A_3 = v_3 N'(v_0) + v_1 v_2 N''(v_0) + \frac{1}{3!}{v_1}^3 N'''(v_0)$

$\vdots$

3.Korteweg-deVries Fifth Order (fKdV) NPD Equation:

The fKdV equation [4] in its general form can be given as,

$$v_t - v_{x^5} = G(x,t,v,v^2,v_x,v_{x^2},v_{x^3})$$

where $v_{x^i} = \frac{\partial^i v}{\partial x^i} = v_{xxx,..,i\ times}$

In particular consider nonlinear partial differential fKdV equation in the form,

$$v_t + v v_x - v v_{xxx} - v_{xxxxx} = 0 \qquad (6)$$

With initial condition $v(x,0) = \exp(x)$

Solution of Nonlinear Partial Differential fKdV By Applying ADM:

In This section we apply ADM for solving nonlinear partial differential fKdV equation.

In operator form equation (6) can be rewritten as,

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$$D_t v = v v_{xxx} - v v_x + D_x v, \quad (7)$$

Apply integral operator D^{-1} to both the sides of equation (7) with $v_0 = e^x$ as,

$$v(x,t) - v(x,0) = D^{-1}(v v_{xxx}) - D^{-1}(v v_x) + D^{-1}(D_x v) \quad (8)$$

Here, $v = \sum_{i=0}^{\infty} v_i$

the components $v_{0,}, v_{1,}, v_2, \ldots$ are obtained by recursive relation.

Also the nonlinear terms $v v_{xxx}$ and $v.v_x$ are obtained by expressing it into infinite series as,

$$v v_{xxx} = \sum_{i=0}^{\infty} A_i \ , \quad v v_x = \sum_{i=0}^{\infty} B_i$$

Where, A_i and B_i are Adomian polynomials.

Hence equation (8) becomes,

$$\sum_{i=0}^{\infty} v_i = e^x + D^{-1}\left(\sum_{i=0}^{\infty} A_i\right) - D^{-1}\left(\sum_{i=0}^{\infty} B_i\right) + D^{-1}(D_x v)$$

Therefore we get recurrence relation,

$$v_{i+1} = D^{-1}\left(\sum_{i=0}^{\infty} A_i\right) - D^{-1}\left(\sum_{i=0}^{\infty} B_i\right) + D^{-1}(D_x v), \qquad i \geq 0$$

Hence we can evaluate,

$$v_1 = D^{-1}(A_0) - D^{-1}(B_0) + D^{-1}(D_x v),$$

$$= \int_0^t v_0 v_{0_{xxx}} \, dt - \int_0^t v_0 v_{0_x} \, dt + \int_0^t v_{0_x} \, dt,$$

$$= e^x t$$

$$v_2 = D^{-1}(A_1) - D^{-1}(B_1) + D^{-1}(D_x v),$$

$$= \int_0^t v_1 v_{1_{xxx}} \, dt - \int_0^t v_1 v_{1_x} \, dt + \int_0^t v_{1_x} \, dt,$$

$$= e^x \frac{t^2}{2}$$

$$v_3 = D^{-1}(A_2) - D^{-1}(B_2) + D^{-1}(D_x v),$$

$$= \int_0^t \left(e^x \frac{t^2}{2}\right)\left(e^x \frac{t^2}{2}\right)_{xxx} dt - \int_0^t \left(e^x \frac{t^2}{2}\right)\left(e^x \frac{t^2}{2}\right)_x dt + \int_0^t \left(e^x \frac{t^2}{2}\right)_x dt,$$

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$$= e^x \frac{t^3}{6}$$

$$= e^x \frac{t^3}{3!}$$

Continuing in this way we can determine other components $v_4, v_5, \ldots$

Therefore,

$$v(x,t) = v_0 + v_1 + v_2 + \cdots$$

$$= e^x + e^x t + e^x \frac{t^2}{2} + e^x \frac{t^3}{3!} + \cdots$$

$$= e^x \left(1 + t + \frac{t^2}{2} + \frac{t^3}{3!} + \cdots \right)$$

$$= e^{x+t} \qquad (9)$$

Hence by using ADM we get solution of nonlinear partial differential fKdV equation (5.28) represented by equation (9).

Also exact solution of fKdV equation (6) is $v = e^{x+t}$ whose graphical representation is shown by Figure,

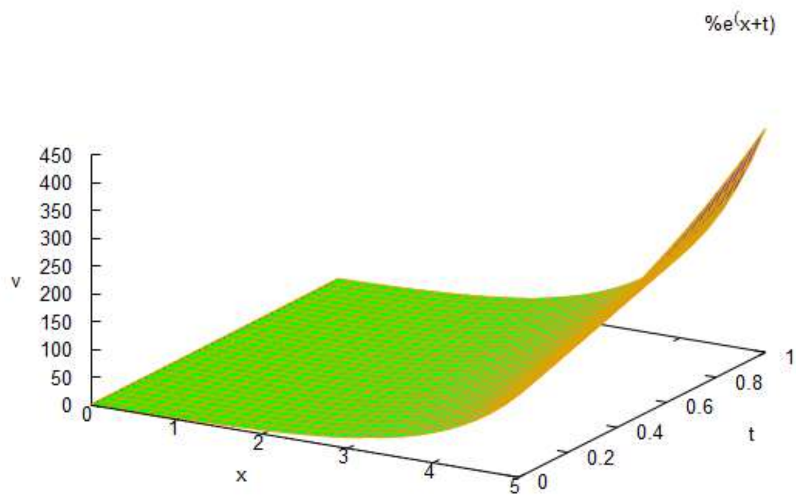

Figure : The Exact solution of fKdV equation (6).

Conclusions:

In this article, ADM is successfully applied for obtaining solution of nonlinear partial differential fKdV equation with initial condition. Results obtained by ADM has good agreement with exact solution.Hence, ADM is more powerful and effective mathematical tool for finding solution of nonlinear partial differential fKdV equation. Hence this method gives better accuracy for fKdV equation.

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