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INDEPENDENT VICT DOMINATION IN GRAPHS

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Abstract:

For any graph , the Vict graphV_n (G) of a graph ,is a graph whose vertex set is the union of the set of vertices and set of cutvertices of in which two vertices are adjacent if and only if the corresponding members are adjacent or incident. A dominating set of a vict graph V_n (G) is an independent dominating set if the induced subgraph $<D>$ has no edges. The minimum cardinality of a minimal independent dominating set is called the Independent Vict Domination Number and is denoted by $i_n(G)$ **.**

In this paper, many bounds on $i_n(G)$ were obtained in terms of the vertices, edges and many other different parameters of but not in terms of the elements of V_n (G) . Further its relation with other different parameters are also developed.

INTRODUCTION:

In this paper, all the graph considered here are simple, finite, nontrivial, undirected and connected. The vertex set and edge set of graph G are denoted by $V(G) = p$ and $E(G) = q$ respectively. Terms not defined here are used in the sense of Harary [2].

The degree, neighborhood and closed neighborhood of a vertex v in a graph G are denoted by $deg(v)$,$N(v)$,and $N[v] = N(v) \cup \{v\}$ respectively. For a subset S Of V,the graph induced by$S \subseteq V$is denoted by$\langle S \rangle$.

As usual, the maximum degree of a vertex (edge) in G is denoted by $\Delta(G)$ ($\Delta'(G)$).For any real number x, $\lceil x \rceil$ denotes the smallest integer not less than x and $\lfloor x \rfloor$ denotes the greater integer not greater than x.

A vertex cover in a graph G is a set of vertices that covers all the edges of G. The vertex covering number $\alpha_o(G)$ is the minimum cardinality of a vertex cover in G. An edge cover of a graph G without isolated vertices is a set of edges of G that covers all the vertices of G. The edge covering number $\alpha_1 (G)$ of a graph G is the minimum cardinality of an edge cover of G. A set of vertices/edges in a graph G is called independent set if no two vertices/edges in the set are adjacent. The vertex independence number $\beta_0 (G)$ is the maximum cardinality of an

independent set of vertices. The edge independence number $\beta_1(G)$ of a graph G is the maximum cardinality of an independent set of edges.
A set D of graph G=(V,E) is called a dominating set if every vertex in

$V - D$ is adjacent to some vertex in D.The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating set of G.
The edge dominating set F is called connected edge dominating set of G, if the induced subgraph $\langle F\rangle$ is also connected. The connected edge dominating set of

G is denoted by $\gamma'(G)$ and is the minimum cardinality of the connected edge dominating set. Edge domination number was studied by S.L.Mitchell and Hedetniemi [4].

A dominating set D of a graph G is a Strong Split dominating set if the induced subgraph $\langle V - D\rangle$ is totally disconnected with only two vertices. The Strong Split domination number $\gamma_{ss}(G)$ of a graph G is the minimum cardinality of a strong split dominating set of G. See [3].
A dominating set D of a graph $G = (V,E)$ is an independent dominating set if the induced subgraph $\langle D\rangle$ has no edges. The independent domination number $i(G)$ of a graph G is the minimum cardinality of an independent dominating set.

A set $D'\subseteq V'$ is said to be dominating set of $n(G)$,if every vertex in $V' - D'$ is adjacent to some vertex in D'. The domination number of $n(G)$ is denoted by $\gamma_n(G)$ and is the minimum cardinality of dominating set in $n(G)$.
Analogously, we define Independent ***Vict Domination Number*** as follows.

A dominating set $D'$of a Vict graph $V_n(G)$ is an independent dominating set if the induced subgraph $\langle D'\rangle$ has no edges. The independent lict domination number
$i_n(G)$ is the minimum cardinality of an independent dominating set of $n(G)$.

In this paper, many bounds on $i_n(G)$ were obtained and expressed in terms of the vertices , edges and other parameters of G but not in terms of members of $n(G)$. Also we establish independent Vict domination number and express the results with other different domination parameters of G.
We need the following Theorems to prove our later results.

Theorem A[1]:For any connected(p,q) graph G, $\gamma'(G) \leq [p/2]$

Theorem B [3]:For any graph G, $\gamma_{ss}(G) = \alpha_o(G) + p_0$ where p_0 is the number of isolated vertices of G.

Theorem C[3]:For any connected graph G, $\gamma_c(G) \le p - \Delta(G)$.

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RESULTS

First we list out the exact values of $i_n(G)$ for some standard graphs.

Theorem1.2.1:

a. For any path P_p with $p \le 2$ vertices,
$i_n(P_p) = p/2 - 1$ if p is even
$i_n(P_p) = p/2$ if p is odd

b. For any cycle C_p,

$i_n(C_p) = [p/3]$

c. For any star $K_{1,p}$,

$i_n(K_{1,p}) = 1$

d. For any Wheel W_p,

$i_n(W_p) = p/2$ where p is even

$i_n(W_p) = [p/2]$ where p is odd

e. For any complete graph $K_{1,p}$,

$i_n(K_p) = [p/2]$

The following Theorem relates $i_n(G)$ and $\gamma_n(G)$ in terms of the edges of (G) .

Theorem1.2.2:For any connected (p,q) graph G, $\gamma_n(G)+ i_n(G) \leq q$.

Proof. Let $D_1 \subseteq V(n(G))=\{v_1,v_2,\ldots\ldots v_i\}$ for each i,$1\leq i \leq n$, be the set of vertices in $n(G)$ such that $N[D_1] = V(n(G))$.Then$|D_1|= \gamma_n(G)$ .Suppose for each $v_i \in D_1$, the induced subgraph $\langle D_1\rangle$ contains the set of vertices such that $v_i = 0$.Then D_1 itself forms an independent dominating set of $n(G)$. Otherwise, let S =

$D_2 \cup I$ where $D_2 \subseteq D_1$ and $I = V(n(G)) - D_1$, such that for all $v_i \in \langle D_2 \cup I\rangle$,

$deg(v_i) = 0$.Thus $\langle D_2\cup I\rangle$forms a minimal independent dominating set of

$n(G)$ .Now since $V(n(G)) = E(G) \cup C(G)$ where $C(G) = \{c_1, c_2, \ldots\ldots c_n\} \subseteq$

$V(n(G))$ is the set of cut vertices in G.It follows that $|D_1|\cup |S| \leq |E(G)|$,which gives

$\gamma_n(G) + i_n(G) \leq q$..

The Theorem below relates $i_n(G)$ and $\gamma_{ss}(G)$

Theorem1.2.3: For any connected non trivial graph G, $i_n(G) \leq \gamma_{ss}(G)$

Proof : Suppose $F = \{e_1, e_2, \ldots\ldots e_j\} \subseteq E(G)$ be the edge dominating set of graph G and $C= \{c_1, c_2, \ldots\ldots c_i\}$ be the set of cut vertices in G. In $n(G)$, since $V(n(G)) = E(G) \cup C(G)$, then the set $D_1 \subseteq F \cup C$ such that $N[D_1] = V(n(G))$ is the minimal dominating set of $n(G)$. Suppose $E(D_1) = \Phi$ in the subgraph $\langle D_1\rangle$, then D_1 itself is an independent dominating set of $n(G)$.Otherwise, let $I = \langle D_2 \cup D'_2\rangle$ where $D_2 \subseteq D_1$ and $D'_2 \subseteq V(n(G)) - D_1$ such that no two vertices in $\langle D_2 \cup D'_2\rangle$ are adjacent. Hence the sub graph $\langle D_2\cup D'\rangle$ forms an independent dominating set of

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$n(G)$. Further let S $\subseteq V(n(G))$ be the maximum independent set of vertices incident on F in G and S' $\subseteq$ S be the set of all isolated vertices in G, then $\langle (V - S) \cup S' \rangle$ forms a strong split domination in G.Thus$|D_2 \cup D'_2| \leq |V - S| \cup S'$ which gives $i_n(G) \leq \gamma_{ss}(G)$.

Theorem1.2.4: For any connected graph G, $i_n(G) \leq \alpha_o(G) + p_0$ where $p_0 = V(G) - \gamma_{ss}(G)$

Proof: From Theorem B and Theorem 3 we have the required result.

Theorem1.2.5:For any connected non-trivial graph G, $i_n(G) \leq \beta_1(G)$.

Proof: Suppose $F \subseteq E(G) = \{e_1, e_2 \ldots\ldots\ldots e_i\}$,$\forall\ 1 \leq i \leq n$ be an edge dominating set of G and $C = \{c_1, c_2, c_3 \ldots\ldots c_n\}$be the set of cut vertices in G .Now suppose J $\subseteq F(G)$ is a set of maximum edges in G ,such that for any $e_i, e_j \in J$, $N[e_i] \cap N[e_j] = \Phi$, $1 \leq I, j \leq n$, then J for ms maximal edge independent set of G with $|J| = \beta_1(G)$. Since $V(n(G)) = E(G) \cup C(G)$,then there exist an independent set of vertices $D \subseteq J' \cup H$ where $J' \subseteq J$ and $H \subseteq V(n(G)) - J$ such that H ø N[J] , which covers all the vertices in $n(G)$.Then clearly $\langle D \rangle$ forms a minimal independent dominating set in $n(G)$. It follows that $|D| \leq |J|$ which gives $i_n(G) \leq \beta_1(G)$.

Theorem1.2.6: For any connected (p,q) graph G, $i_n(G) \leq (p/2)$

Proof: Let $S = \{e_1, e_2 \ldots\ldots\ldots\ldots e_n\}$be the minimal edge dominating set in G and

$C = \{c_1, c_2 \ldots\ldots\ldots\ldots c_n\}$be the set of cut vertices in G.Then $S \cup C \subseteq V(n(G))$. Now, we consider a minimal set of vertices $D_1 \subseteq S \cup C$ in $n(G)$ such that $N[D_1] = V(n(G))$. Then D_1 is the minimal dominating set in $n(G)$. Further if $E(D_1) = \Phi$ in the subgraph $\langle D_1 \rangle$, then D_1 itself is an independent dominating set of $n(G)$.Otherwise, let $I = D_2 \cup D'_2$ where $D_2 \subseteq D_1$ and $D'_2 \subseteq V(n(G)) - D_1$ such that no two vertices in

$\langle D_2 \cup D'_2 \rangle$ are adjacent. Hence the subgraph $\langle D_2 \cup D'_2 \rangle$ forms an independent

dominating set of $n(G)$. Since each $e_1 \in S$ is incident on two vertices and also by

Theorem A, we get

$$i_n(G) \leq (p/2)$$

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